FURTHER MATHEMATICS **9231/02**

Paper 2

October/November 2003

3 hours

Additional materials: Answer Booklet/Paper
                         Graph paper
                         List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

This document consists of **7** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

[Turn over

1 Write down the moment of inertia of a uniform square lamina $ABCD$, with side $2a$ and mass M, about an axis through its centre perpendicular to its plane. [1]

Show that the moment of inertia of the lamina about the diagonal AC is $\frac{1}{3}Ma^2$. [2]

Deduce the moment of inertia about the hypotenuse PR of a uniform isosceles right-angled triangular lamina PQR of mass m, in which $PR = b$. [3]

2 An empty shopping trolley E of mass 2 kg is moving at a speed of 2.5 m s^{-1} on a smooth horizontal floor towards a full trolley F of mass 30 kg. Just before they collide, F is moving at 1.5 m s^{-1} towards E. The collision is modelled as a direct impact between two particles with coefficient of restitution 0.2. Find the speed and the direction of motion of E immediately after the collision. [5]

During the impact the trolleys are in contact for 0.1 s, and the force between them is constant. Find the magnitude of this force. [2]

3
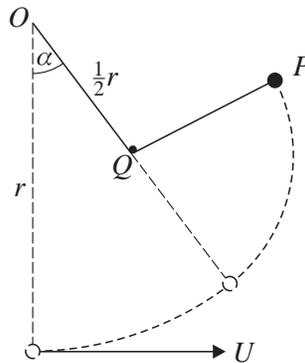
A particle P of mass m is connected to a fixed point O by a light inextensible string OP of length r, and is moving in a vertical circle, centre O. At its lowest point, P has speed U. When the string makes an angle α with the downward vertical it encounters a small fixed peg Q, where $OQ = \frac{1}{2}r$. The string proceeds to wrap itself around the peg, so that P begins to move in a vertical circle with centre Q (see diagram). Given that the particle describes a complete circle about Q, show that

$$U^2 \geqslant gr\left(\tfrac{7}{2} - \cos \alpha\right).$$ [7]

4
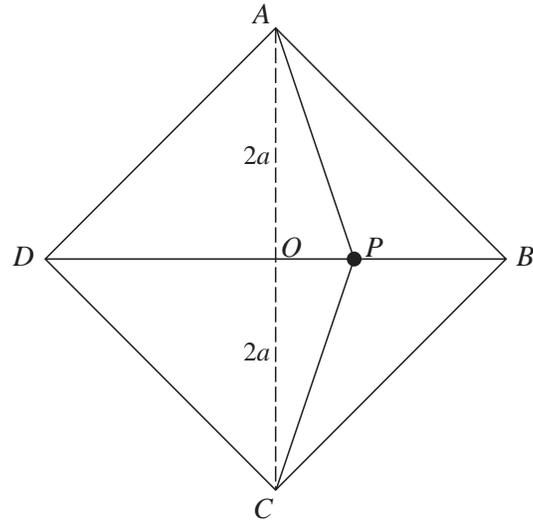
ABCD is a smooth square horizontal table with centre *O* and diagonals of length $4a$. A particle *P* of mass *m* rests on the table and is connected to *A* and to *C* by two light elastic strings, each of natural length *a* and modulus of elasticity λ (see diagram). Show that, when *P* is displaced a short distance *x* from *O* towards *B*, the resultant force towards *O* is $\dfrac{\lambda x}{a}$, if $\left(\dfrac{x}{a}\right)^2$ and higher powers of $\dfrac{x}{a}$ are neglected. [5]

Two additional strings, *PB* and *PD*, are now introduced, so that *P* is connected to all four corners of the table by strings of natural length *a* and modulus of elasticity λ. The particle is displaced from *O* a short distance towards *B*, and is released from rest. Show that *P* describes approximate simple harmonic motion along *DB*, and find the period of this motion. [6]

5
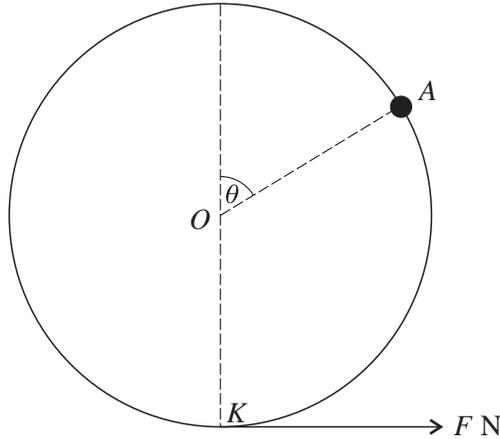
In an amusement park, a big wheel of radius 6 m turns freely about a horizontal axis at its centre O and carries passengers in cabins fixed symmetrically around its circumference. When there are no passengers, the centre of mass of the wheel and cabins is at O, and the moment of inertia of the system about its axis is $10\,000\,\text{kg m}^2$. On a certain ride, only one cabin, A, contains passengers. At time t seconds the radius to A makes an angle θ with the upward vertical, as shown in the diagram. The wheel is at rest when $\theta = \frac{1}{3}\pi$, and the operator turns off the engine and controls the ride by means of a brake at the lowest point K, where there is a constant horizontal force of magnitude F newtons, tangential to the wheel. The passengers are modelled as a particle of mass 100 kg at A.

  (i) Show that $6800\ddot{\theta} = 300g\sin\theta - 3F$. [3]

 (ii) By using the fact that $\ddot{\theta} = \omega\dfrac{\mathrm{d}\omega}{\mathrm{d}\theta}$, where $\omega = \dot{\theta}$, or otherwise, show that

$$6800\omega^2 = 300g(1 - 2\cos\theta) + 2F(\pi - 3\theta).$$ [3]

(iii) Find the value of F for which the wheel will come to rest when A reaches K. Find also the greatest angular speed during this ride. [6]

6 A random sample of 10 observations of a normal variable X has mean $\bar{x}$, where

$$\bar{x} = 4.344, \quad \sum(x - \bar{x})^2 = 0.8022.$$

Carry out a 2-tail test, at the 5% level of significance, to test whether the mean of X is 4.58. State your null and alternative hypotheses clearly. [7]

7 The strength of concrete depends on the method used for drying it. Random samples of 4 strengths after using method A, and of 6 strengths after using method B, are given in the table, in suitable units.

Strength after A: 3100, 3230, 3320, 3350.

Strength after B: 3150, 3280, 3400, 3500, 3520, 3550.

Test whether the two methods produce concrete with different mean strengths, using a 5% significance level. You may assume that the two populations are normal with equal variances. [7]

8 A naturalist is observing the numbers of birds alighting on a certain field in the course of 200 separate minutes. The results are shown in the table.

Number of birds	0	1	2	3	4	5	6	7 or more
Observed frequency	27	46	65	39	12	7	4	0

The naturalist claims that a Poisson distribution fits the data. Fit a Poisson distribution to the data, and test the naturalist's claim at the 5% level of significance. [8]

9 Alec and Bill take alternate turns at kicking a football at a goal, and their probabilities of scoring a goal on each kick are p_1 and p_2 respectively, independently of previous outcomes. The first person to score allows the other person one more kick. If the other then scores, the game is drawn. If the other then misses, the first has won the game. Alec begins a game. Show that the probability that

(i) Alec scores first, on his nth kick, is $p_1(q_1q_2)^{n-1}$, where $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$, [2]

(ii) Alec wins the game is $\dfrac{p_1q_2}{1 - q_1q_2}$, [3]

(iii) the game is drawn is $\dfrac{p_1p_2(1 + q_1)}{1 - q_1q_2}$. [3]

Given that $p_1 = p_2 = \frac{1}{3}$, find the expected total number of kicks in the game. [2]

10 A metalworker produces cubes of various sizes. The length, X cm, of the edge of a randomly chosen cube has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}x & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

The volume of a randomly chosen cube is Y cm^3. By first finding $P(X \leqslant x)$, show that

$$P(Y \leqslant y) = \tfrac{1}{8}(y^{\frac{2}{3}} - 1) \quad \text{for } 1 \leqslant y \leqslant 27.$$ [4]

Hence find $E(Y)$ and $Var(Y)$. [3]

A random sample of 40 cubes has mean volume 10.0 cm^3 and unbiased variance estimate 45 cm^6. Obtain a 95% confidence interval for the mean volume of the cubes produced by the metalworker. [3]

Give a possible reason why the interval you have found may not contain the population mean. [1]

11 Answer only **one** of the following two alternatives.

EITHER
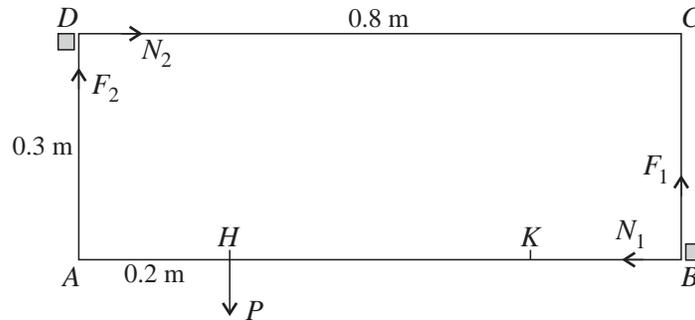
A rectangular drawer $ABCD$, where $AD = 0.3$ m, $DC = 0.8$ m, can move on a smooth horizontal surface if its two handles at H and K are pulled horizontally perpendicular to its front edge AB. An attempt is made to open the drawer by pulling on only one handle H, where $AH = 0.2$ m. In this case, frictional forces F_1 and F_2, at B and D respectively, from the sides of the drawer frame impede opening (see diagram). The normal reactions at B and D are N_1 and N_2 respectively.

 (i) Assuming that the coefficient of friction at B is 0.8, and that the contact at D is smooth, show that the drawer cannot be opened by a force P applied to H. [4]

(ii) Assume now that $\dfrac{F_1}{N_1} = \dfrac{F_2}{N_2}$ at all times, and that the coefficients of friction at B and D are each equal to μ.

    (a) Find the value that μ must exceed if the drawer cannot be opened by a force P applied to H. [4]

    (b) An attempt is made to open the drawer by applying at H, in addition to the force P, a horizontal force Q along the front edge. The reactions of the sides of the drawer frame on the drawer still act at B and D. Given that $\mu = 0.8$, and that the drawer is on the point of moving, find the direction of Q and the ratio of the magnitude of Q to the magnitude of P. [6]

[The second alternative for Question 11 is printed on the next page.]

OR

A random sample of eight pairs of values of x and y is used to obtain the following equations of the regression lines of y on x and of x on y respectively.

$$y = -\frac{7}{10}x + \frac{151}{10}, \qquad x = -\frac{7}{6}y + 20.$$

Seven of the pairs of data are given in the table.

x	10	11	12	11	17	14	19
y	9	8	7	6	5	4	1

Find the eighth pair of values of x and y. [5]

Determine the value of the product moment correlation coefficient, and say what it leads you to expect about the scatter diagram for this sample. [4]

Let Y be the value obtained by substituting a sample value of x into the equation of the regression line of y on x. Evaluate Y for each of the eight values of x and verify that

$$\sum(y - Y)^2 = 8.8.$$ [3]

For each of the eight sample values of x, Y' is given by $Y' = a + bx$, where a and b are any constants. What can you say about the value of $\sum(y - Y')^2$? [2]

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